The book starts with a short survey of the theory of orthogonal polynomials of one variable, with special emphasis on the classical orthogonal polynomials. The second chapter contains some basic examples of orthogonal polynomials of several variables, such as spherical harmonics and the classical orthogonal polynomials on the ball and simplex. These examples are very convenient to have in mind when reading the third chapter, which deals with the general theory of orthogonal polynomials of several variables. In this chapter a "basis independent" theory of orthogonal polynomials of several variables is developed, which in particular leads to analogs of the three-term recurrence relation and Favard's theorem.

The remaining chapters of the book mainly deal with explicit examples of orthogonal polynomials of several variables with respect to reflection-invariant measures. In all these examples an essential role is played by Dunkl's rational differential-reflection operators (which generalize the partial differentiation operators $\partial/\partial x_i$).

Chapter 4 contains a short introduction to finite reflection groups and an introduction to Dunkl's differential-reflection operators. Furthermore, the intertwining operator (which maps $\partial/\partial x_i$ to the associated Dunkl operator) and the generalized exponential function are introduced. The sum of the squares of the Dunkl operators generalizes the Laplace operator and thus naturally leads to the theory of generalized spherical harmonics (called h-harmonics in the book). This topic is treated in Chapter 5, as well as the related Poisson kernels. At the end of the chapter analogs of the Fourier transform are discussed.

Orthogonal polynomials of several variables are considered to be classical if they satisfy a second-order differential-reflection equation. In Chapter 6 some examples of classical orthogonal polynomials of several variables are discussed, such as the generalized Hermite and Laguerre polynomials, and generalized classical orthogonal polynomials on the ball and simplex. Chapter 7 deals with the summability of orthogonal expansions for the h-harmonics and for the generalized classical orthogonal polynomials.

In Chapter 8 the (non)symmetric Jack polynomials are studied with respect to several different scalar products. The Dunkl operators associated with the symmetric group play an essential role in constructing a large family of self-adjoint, commuting operators for which the (non)symmetric Jack polynomials are joint eigenfunctions. The extension of the theory to the setting of the octahedral groups is discussed in Chapter 9.

The authors have restricted their attention to the orthogonal polynomials of several variables associated with rational Dunkl operators. In particular, the generalized Jacobi polynomials of Heckman and Opdam, which are associated with trigonometric-type Dunkl operators, are not discussed. Furthermore, q-deformations of the orthogonal polynomials (such as the Macdonald polynomials) are not treated in the book. The connection of orthogonal polynomials of several variables with completely integrable quantum many-body systems is the only application which is discussed in detail.

This book is a valuable addition to the literature, especially since it is the first detailed and modern treatment on the theory of orthogonal polynomials of several variables. It is reasonably self-contained and carefully written. There are several typos in the book, but most of them are harmless. The book is also well suited for non-specialists. Undoubtedly it will be very useful to anyone interested in orthogonal polynomials of several variables.

Jasper V. Stokman
E-mail: `jstokman@science.uva.nl`
doi:10.1006/jath.2001.3602

Gary D. Knott, *Interpolating Cubic Splines*, Progress in Computer Science and Applied Logic **18**, Birkhäuser, Boston, 2000, xii + 244 pp.

A book with a mathematical topic is generally a text that a reader (or some reader) should be able to learn something from. If it is a good book, then the learning should also be connected with some joy. How about the book under review?

In the first place the very restrictive title is conspicuous. There is the restriction to *interpolating* and to *cubic* splines. The sheer use of cubic splines is justified in the preface (p. x): *"Splines based on higher-degree [than three] polynomials are infrequently found in practical applications and their study adds only limited theoretical content..."* However, lower degree splines, in particular linear splines, play a very important role in graphing. They are omitted from the text. Also in the preface (p. xi) the author continues: *"This text does not use the classic structure consisting of theorems and proofs in sequence, ... Rather a discursive style is employed."*

The book is separated into 18 chapters, the titles of which are as follows:

1. Mathematical Preliminaries (29 pages, 94 exercises scattered in the text).
2. Curves (19, 56).
3. Surfaces (7, 10).
4. Function and Space Curve Interpolation (3, 3).
5. 2D-Function Interpolation (11, 18).
6. *Λ*-Spline Curves With Range Dimension *d* (2, 2).
7. Cubic Polynomial Space Curve Splines (17, 42).
8. Double Tangent Cubic Splines (6, 6).
9. Global Cubic Space Curve Splines (21, 29).
10. Smoothing Splines (10, 17).
11. Geometrically Continuous Cubic Splines (6, 7).
12. Quadratic Space Curve Based Cubic Splines (4, 12).
13. Cubic Spline Vector Space Basis Functions (13, 17).
14. Rational Cubic Splines (2, 0).
15. Two Spline Programs (33, 0).
16. Tensor Product Surface Splines (17, 14).
17. Boundary Curve Based Surface Splines (6, 5).
18. Physical Splines (16, 19).

Let us have a look at some parts of the text in closer detail. In Chapter 2 on curves, we find (p. 31): *"A plane curve is a mapping from some interval* $[a, b] \subset \mathbf{R}$ *into* $\mathbf{R}^2$. *The domain of the curve mapping* $[a, b]$ *may be open, half open, or closed... The parametric representation of a curve is not unique."* A statement when two curves are identical is, however, not given. On p. 38 there is a little section titled *Arc Length Parameterization.* It begins as follows: *"The length of a curve is called its* arc length. *A curve is called* rectifiable *if its arc length exists."* Almost immediately after this we find Exercise 2.24: *Show that the length of the rectifiable curve x, starting at the point* $x(l)$ *and ending at the point* $x(h)$, *is* $\int_{l<t<h} |x'(t)|$. How should a reader be able to solve this exercise with the above definition?

If we take a look at the short Chapter 4 on *Function and Space Curve Interpolation*, we find a loose description of the problem including interpolation with prescribed directions. There is a literal description of what might be desirable, but not more. In Chapter 5 on *2D-Function, Interpolation* some explicit results appear: the Lagrange interpolation formula, and a formula for an odd number of equidistantly distributed abscissae, called Whittaker's formula. If one wants to read a little about the background of that formula one should read de Boor's book [1978, p. 236] which is listed as DeB78. The explanation given by the author based on Fourier transforms is not comprehensible in the context of the book under review. A formula for cubic splines is given where the values and slopes at the breakpoints are prescribed, and some methods are mentioned for estimating the slopes by various means (e.g., Akima's method). There are also some hints on finding monotone interpolating cubic splines, in case the data are monotone, by imposing certain conditions on the slopes. Some error estimates are quoted from de Boor (1). A method for finding cubic splines only from values at the breakpoints is postponed to Chapter 7.

In Chapter 7 on *Cubic Polynomial Space Curve Splines* there is a cubic interpolation formula for connecting two points in $\mathbf{R}^3$ with prescribed slopes (also in $\mathbf{R}^3$). The question of how to find such a formula is, however, not raised. Most of the material of this section is hidden in exercises. At the end Bernstein–Bézier curves are mentioned and the fact that they do not interpolate the control points (apart from the first and the last one) is emphasized.

Chapter 9 on *Global Cubic Space Curve Splines* is, apart from the beginning chapter and Chapter 15 with two C programs, the longest. It contains the usual derivations of cubic splines (interpolating at the breakpoints) as solution of tridiagonal linear systems. Minimal properties are also given. It is not quite clear why at this point long derivations occur where at other places just results are given.

We could continue refereeing all further chapters and some details might be of interest. However, there is some doubt in general whether "the discursive style employed" is of much use for a greater audience. There are several figures in the book, all of them untitled. This makes it occasionally difficult to find the corresponding connection with the text. There are no worked numerical examples at all. The references (three pages) range in time from 1946 to 1995. Most (but not all) of the quoted authors are listed by first and last name. In general, exercises are welcome in a book, but here, where the many exercises are distributed over the whole text (sometimes including solutions), they disturb the fluent reading of the material. The idea of always insisting on interpolation at prescribed points is questionable. Good approximations to curves (or surfaces) generally also interpolate at some points which are not known in advance.

To answer the question from the beginning of this review: it might be possible to learn something from this book, but without any joy. A mathematically trained person should better consult other books.

REFERENCES

1. C. de Boor, "A Practical Guide to Splines," Springer-Verlag, New York/Heidelberg/Berlin, 1978.

Gerhard Opfer
E-mail: opfer@math.uni-hamburg.de
doi:10.1006/jath.2001.3603

Gheorghe Micula and Sanda Micula, *Handbook of Splines*, Mathematics and its Applications **462**, Kluwer, Dordrecht, 1999, xvi + 604 pp.

Splines play a fundamental role in applied mathematics and in various fields of applications. In the last 50 years, many thousands of papers on splines were published. The reason is that splines are efficient tools for solving a variety of problems because they possess nice structural properties and have excellent approximation powers and fast algorithms for computing splines are available (although the important multivariate problems are not solved completely). The aim of this book is to give an introduction to the theory of splines together with applications to various fields such as data fitting, interpolation, approximation, computer aided design, integral equations, differential equations, stochastics, and wavelets.

Chapter 1 contains well-known results on B-spline bases and on interpolation by natural, discrete, and periodic splines. Moreover, the relation of splines and the optimal approximation of linear functionals is investigated. In Chapter 2 the Bézier representation, the dimension